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ROUGH BIPOLAR NEUTROSOPHIC SET

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ABSTRACT
Bipolar neutrosophic set theory and rough neutrosophic set theory are emerging as powerful tool for dealing with uncertainty, and indeterminate, incomlete, and inprecise information. In the present study we develop a hybrid structure called "rough bipoar neutrsophic set". In the study, we define rough bipoar neutrsophic set and define union, complement, intersection and containment of rough bipolar neutrosophic sets.

INTRODUCTION
The notion of fuzzy set which is non-statistical in nature was introduced by Zadeh in 1965[1] to deal with uncertainty. Fuzzy set has been applied in many real applications to handle uncertainty. In 1986, Atanassov [2] extended the concept of fuzzy set to intuitionistic fuzzy set by introducing the degree non-membership as an independent component. In 1998, Smarandache [3] grounded the concept of degree of indeterminacy as independent component and defined neutrosophic set. In 2005, Wang et al. [4] introduced the concept of single valued neutrosophic set (SVNS) which is an instance of a neutrosophic set to deal real scientific and engineering applications.

Lee [5] introduced the concept of bipolar fuzzy sets, as an extension of fuzzy sets. In bipolar fuzzy sets the degree of membership is extended from [0, 1] to [-1, 1]. In a bipolar fuzzy set, if the degree of membership of an element is zero, then we say the element is unrelated to the corresponding property, the membership degree (0, 1] of an element specifies that the element somewhat satisfies the property, and the membership degree [−1, 0) of an element implies that the element somewhat satisfies the implicit counter-property [6].

In 2014, Broumi et al. [7, 8] presented the concept rough neutrosophic set to deal indeterminacy in more flexible way. Pramanik and Mondal [9, 10] and Mondal and Pramanik [11, 12, 13] studied different applications of rough neutrosophic sets in decision making. Deli et al. [14] defined bipolar neutrosophic set and showed numerical example for multi-criteria decision making problem.

In this paper we combine bipolar neutrosophic set and rough neutrosophic set and define rough bipolar neutrosophic set. We define the union, complement, intersection and containment of rough bipolar neutrosophic sets.

The rest of the paper has been organized as follows. Section 2 presents mathematical preliminaries of fuzzy bipolar set, neutrosophic set, single valued neutrosophic set, bipolar neutrosophic set, and rough neutrosophic set. Section 3 is devoted to define rough bipolar neutrosophic set.

SOME RELEVANT DEFINITIONS
In this section we recall some basic definitions of bipolar valued fuzzy set, neutrosophic set, single valued neutrosophic sets and rough neutrosophic set.

Definition 2.1 Bipolar valued fuzzy set [5]

Let U be the universe of discourse. Then a bipolar valued fuzzy set B on U is defined by positive membership function α_B^+, i.e. α_B^+: $U \to [0,1]$ and a negative membership function α_B^-, i.e.

$\alpha_B^-: U \to [-1, 0]$.

Mathematically a bipolar valued fuzzy set is represented by B = $\langle z : (\alpha_B^+(z), \alpha_B^-(z)) \rangle$ $\forall z \in U$

Definition 2.2 Neutrosophic set [3]

Let U be the universe of discourse. Then a neutrosophic set S in U with generic elements x is characterized by a truth membership function $T_S(x)$, an indeterminacy membership function $I_S(x)$ and a falsity membership function $F_S(x)$. There is no restriction on $T_S(x)$, $I_S(x)$ and $F_S(x)$ other than they are subsets of $]^-0, 1^+[$ that is $T_S(x)$: $N \to\ ]^-0, 1^+[$; $I_S(x)$: $N \to\ ]^-0, 1^+[$; $F_S(x)$: $N \to\ ]^-0, 1^+[$

Therefore, $^-0 \le \inf\ T_S(X) + \inf\ I_S(X) + \inf\ F_S(X) \le \sup T_S(X) + \sup I_S(X) + \sup F_S(X) \le 3^+$.

Definition 2.3 Single-valued neutrosophic set [4]

Let U be a universal space of points (objects) with a generic element of U denoted by x.

A single valued neutrosophic set S is characterized by a truth membership function $T_S(x)$, a falsity membership function $F_S(x)$ and indeterminacy function $I_S(x)$ with $T_S(x)$, $F_S(x)$, $I_S(x)$ $\in$ $[0,1]$ for all x in U.

When U is continuous, a SNVS S can be written as follows:

$S = \int_x \langle T_S(x), I_S(x), F_S(x) \rangle / x, \forall x \in U$ and when U is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(x), I_S(x), F_S(x) \rangle / x, \forall x \in U$$

There is no restriction on $T_S(x)$, $I_S(x)$ and $F_S(x)$ other than they are subsets of [0, 1] that is

$T_S(x)$: $N \to [0, 1]$; $I_S(x)$: $N \to [0, 1]$; $F_S(x)$: $N \to [0, 1]$

Therefore, $0 \le \inf\ T_S(X) + \inf\ I_S(X) + \inf\ F_S(X) \le \sup T_S(X) + \sup I_S(X) + \sup F_S(X) \le 3$

Definition 2.4 bipolar neutrosophic set [14]

A bipolar neutrosophic set A in Z is defined as an object of the form

A={< $T^+(z)$, $I^+(z)$, $F^+(z)$, $T^-(z)$, $I^-(z)$, $F^-(z)$ >: z ∈ Z}, where ,T^+, I^+, F^+ : $X \to [0, 1]$ and , T^-, I^-, F^-: $X \to [-1, 0]$. The positive membership degree $T^+(z)$, $I^+(z)$, and $F^+(z)$ denote the truth membership, indeterminate membership and false membership respectively of an element z ∈ Z corresponding to a bipolar neutrosophic set A. The negative membership degree $T^-(z)$, $I^-(z)$, and $F^-(z)$ denote the truth membership, indeterminate membership and false membership respectively of an element z ∈ Z to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Definition 2.5 [14] Let, B_1 = {x, $\langle T_{B_1}^+(x), I_{B_1}^+(x), F_{B_1}^+(x), T_{B_1}^-(x), I_{B_1}^-(x), F_{B_1}^-(x) \rangle$ | x∈ U} and B_2 = {x, $\langle T_{B_2}^+(x), I_{B_2}^+(x), F_{B_2}^+(x), T_{B_2}^-(x), I_{B_2}^-(x), F_{B_2}^-(x) \rangle$ | x∈ U} be two BNSs. Then $B_1 \subseteq B_2$ if and only if

$T_{B_1}^+(x) \le T_{B_2}^+(x)$, $I_{B_1}^+(x) \le I_{B_2}^+(x)$, $F_{B_1}^+(x) \ge F_{B_2}^+(x)$; $T_{B_1}^-(x) \ge T_{B_2}^-(x)$, $I_{B_1}^-(x) \ge I_{B_2}^-(x)$, $F_{B_1}^-(x) \le F_{B_2}^-(x)$ for all x∈ U.

Definition 2.6 [14] Assume that B_1 = {x, $\langle T_{B_1}^+(x), I_{B_1}^+(x), F_{B_1}^+(x), T_{B_1}^-(x), I_{B_1}^-(x), F_{B_1}^-(x) \rangle$ | x∈ U} and B_2 = {x, $\langle T_{B_2}^+(x), I_{B_2}^+(x), F_{B_2}^+(x), T_{B_2}^-(x), I_{B_2}^-(x), F_{B_2}^-(x) \rangle$ | x∈ U} be two BNSs. Then $B_1 = B_2$ if and only if

$T_{B_1}^+(x) = T_{B_2}^+(x)$, $I_{B_1}^+(x) = I_{B_2}^+(x)$, $F_{B_1}^+(x) = F_{B_2}^+(x)$; $T_{B_1}^-(x) = T_{B_2}^-(x)$, $I_{B_1}^-(x) = I_{B_2}^-(x)$, $F_{B_1}^-(x) = F_{B_2}^-(x)$ for all $x \in U$.

Definition 2.7 [14] Assume that B = {x, $\langle T_B^+(x), I_B^+(x), F_B^+(x), T_B^-(x), I_B^-(x), F_B^-(x)\rangle$ | x∈ U} be a BNS. The complement of B is denoted by B^c and is defined by

$T_{B^c}^+(x) = \{1^+\} - T_B^+(x)$, $I_{B^c}^+(x) = \{1^+\} - I_B^+(x)$, $F_{B^c}^+(x) = \{1^+\} - F_B^+(x)$;

$T_{B^c}^-(x) = \{1^-\} - T_B^-(x)$, $I_{B^c}^-(x) = \{1^-\} - I_B^-(x)$, $F_{B^c}^-(x) = \{1^-\} - F_B^-(x)$ for all x∈ U.

Definition 2.8 [14] Assume that B_1 = {x, $\langle T_{B_1}^+(x), I_{B_1}^+(x), F_{B_1}^+(x), T_{B_1}^-(x), I_{B_1}^-(x), F_{B_1}^-(x)\rangle$ | *x*∈ *U*} and B_2 = {*x*, $\langle T_{B_2}^+(x), I_{B_2}^+(x), F_{B_2}^+(x), T_{B_2}^-(x), I_{B_2}^-(x), F_{B_2}^-(x)\rangle$ | *x*∈ *U*} be two BNSs. Then their union $B_1 \cup B_2$ is defined as follows:

$B_1 \cup B_2$ = {Max $(T_{B_1}^+(x), T_{B_2}^+(x))$, $\frac{I_{B_1}^+(x) + I_{B_2}^+(x)}{2}$, Min $(F_{B_1}^+(x), F_{B_2}^+(x))$, Min $(T_{B_1}^-(x), T_{B_2}^-(x))$, $\frac{I_{B_1}^-(x) + I_{B_2}^-(x)}{2}$, Max $(F_{B_1}^-(x), F_{B_2}^-(x))$} for all *x*∈ *U*.

Definition 2.9 [14] Assume that B_1 = {x, $\langle T_{B_1}^+(x), I_{B_1}^+(x), F_{B_1}^+(x), T_{B_1}^-(x), I_{B_1}^-(x), F_{B_1}^-(x)\rangle$ | x∈ U} and B_2 = {x, $\langle T_{B_2}^+(x), I_{B_2}^+(x), F_{B_2}^+(x), T_{B_2}^-(x), I_{B_2}^-(x), F_{B_2}^-(x)\rangle$ | x∈ U} be two BNSs. Then their intersection $B_1 \cap B_2$ is defined as follows:

$B_1 \cap B_2$ = {Min $(T_{B_1}^+(x), T_{B_2}^+(x))$, $\frac{I_{B_1}^+(x) + I_{B_2}^+(x)}{2}$, Max $(F_{B_1}^+(x), F_{B_2}^+(x))$, Max $(T_{B_1}^-(x), T_{B_2}^-(x))$, $\frac{I_{B_1}^-(x) + I_{B_2}^-(x)}{2}$, Min $(F_{B_1}^-(x), F_{B_2}^-(x))$}for all x∈ U.

Definition 2.10: Definitions of rough neutrosophic set [7, 8]
Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ can be respectively defined as follows:

$\underline{N}(P) = \langle <x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x)> / z \in [x]_R, x \in Z\rangle$,

$\overline{N}(P) = \langle <x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x)> / z \in [x]_R, x \in Z\rangle$

Where, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(z)$, $I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(z)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(z)$,

$T_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$, $I_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$, $F_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, I_P(z)$

So, $0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$ and $0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

Where $\vee$ and $\wedge$ denote "max" and "min'' operators respectively, $T_P(z)$, $I_P(z)$ and $F_P(z)$ are the membership, indeterminacy and non-membership of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mapping $\underline{N}$, $\overline{N}$: N(Z)→ N(Z) are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of R if $\underline{N}(P) = \overline{N}(P)$; i.e. $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$, $F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x)$ for all x belongs to Z.

ROUGH BIPOLAR NEUTROSOPHIC SETS (RBNS)

In this section we introduce the notion of rough bipolar neutrosophic sets by combining rough neutrosophic set and bipolar neutrosophic set. We define union, complement, intersection and containment of rough bipolar neutrosophic sets.

Definition 3.1: Rough bipolar neutrosophic sets

Let Z be a non-null set and R be an equivalence relation on Z. Let B be a bipolar neutrosophic set in Z. The positive membership degrees $T^+(z)$, $I^+(z)$, and $F^+(z)$ respectively denote the truth membership, indeterminate membership and falsity membership respectively of an element z ∈ Z corresponding to a bipolar neutrosophic set B. The negative membership degrees $T^-(Z), I^-(Z)$, and $F^-(Z)$ denote the truth membership, indeterminate membership and false membership of an element z ∈ Z to some implicit counter-property corresponding to a bipolar neutrosophic set B. The lower and the upper approximations of B in the approximation (Z, R) denoted by $\underline{N}(B)$ and $\overline{N}(B)$ are respectively defined as follows:

$$\underline{N}(B) = \left\langle < x,\ T^+_{\underline{N}(B)}(x), I^+_{\underline{N}(B)}(x),\ F^+_{\underline{N}(B)}(x), T^-_{\underline{N}(B)}(x), I^-_{\underline{N}(B)}(x),\ F^-_{\underline{N}(B)}(x) > / \, z \in [x]_R, x \in Z \right\rangle,$$

$$\overline{N}(B) = \left\langle < x,\ T^{\pm}_{\overline{N}(B)}(x), T^{\pm}_{\overline{N}(B)}(x), T^{\pm}_{\overline{N}(B)}(x), T^{-}_{\overline{N}(B)}(x), T^{-}_{\overline{N}(B)}(x), T^{-}_{\overline{N}(B)}(x) > / \, z \in [x]_R, x \in Z \right\rangle$$

Here, $T^+_{\underline{N}(B)}(x) = \wedge_z \in [x]_R\ T^+(z)$, $I^+_{\underline{N}(B)}(x) = \wedge_z \in [x]_R\ I^+(z)$, $F^+_{\underline{N}(B)}(x) = \wedge_z \in [x]_R\ F^+(z)$,

$T^-_{\underline{N}(B)}(x) = \wedge_z \in [x]_R\ T^-(z)$, $I^-_{\underline{N}(B)}(x) = \wedge_z \in [x]_R\ I^-(z)$, $F^-_{\underline{N}(B)}(x) = \wedge_z \in [x]_R\ F^-(z)$;

$T^{\pm}_{\overline{N}(B)}(x) = \vee_z \in [x]_R\ T^+(z)$, $I^{\pm}_{\overline{N}(B)}(x) = \vee_z \in [x]_R\ I^+(z)$, $F^{\pm}_{\overline{N}(B)}(x) = \vee_z \in [x]_R\ F^+(z)$,

$T^{-}_{\overline{N}(B)}(x) = \vee_z \in [x]_R\ T^-(z)$, $I^{-}_{\overline{N}(B)}(x) = \vee_z \in [x]_R\ I^-(z)$ $F^{-}_{\overline{N}(B)}(x) = \vee_z \in [x]_R\ F^-(z)$.

Here $\vee$ and $\wedge$ denote "max" and "min'' operators respectively, the positive membership degrees $T^+(z)$, $I^+(z)$, and $F^+(z)$ denote respectively the degree of truth membership, indeterminate membership and falsity membership of an element z ∈ Z corresponding to a bipolar neutrosophic set B. The negative membership degrees $T^-(z)$, $I^-(z)$, and $F^-(z)$ denote respectively the degrees of truth membership, indeterminate membership and falsity membership of an element z ∈ Z to some implicit counter-property corresponding to a bipolar neutrosophic set B. It is easy to see that $\underline{N}(B)$ and $\overline{N}(B)$ are two rough bipolar neutrosophic sets in Z.

Thus NS mappings $\underline{N}$, $\overline{N}$: N(Z) → N(Z) are, respectively, referred to as the lower and upper rough bipolar NS approximation operators, and the pair $(\underline{N}(B), \overline{N}(B))$ is called the rough bipolar neutrosophic set in (Z, R).

From the above definition, it is seen that $\underline{N}(B)$ and $\overline{N}(B)$ have constant membership on the equivalence classes of R if $\underline{N}(B) = \overline{N}(B)$; i.e. $T_{\underline{N}(B)}(x) = T_{\overline{N}(B)}(x)$, $I_{\underline{N}(B)}(x) = I_{\overline{N}(B)}(x)$, $F_{\underline{N}(B)}(x) = F_{\overline{N}(B)}(x)$ $\forall x \in X$.

Example 3.1

Let X={x_1, x_2, x_3}

$$A = \left\langle \begin{matrix} < x_1\ \{(0.50, 0.30, 0.10, -0.60, -0.40, -0.08), (0.60, 0.40, 0.20, -0.50, -0.30, -0.12)\} > \\ < x_2\ \{(0.30, 0.50, 0.30, -0.40, -0.40, -0.18), (0.60, 0.10, 0.20, -0.30, -0.30, -0.10)\} > \\ < x_3\ \{(0.50, 0.10, 0.30, -0.25, -0.15, -0.20), (0.40, 0.40, 0.10, -0.35, -0.45, -0.20)\} > \end{matrix} \right\rangle$$

Example 3.2

Let Z = {P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8} be the universe of discourse. Let R be an equivalence relation, where its partition of Z is given by

Z/R ={(P_1, P_4), (P_2, P_3, P_6), (P_5), (P_7, P_8)}

Let N(P) = {<P_1(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), P_4(0.3, 0.5, 0.4, -0.3, -0.3, -0.2), P_5(0.4, 0.6, 0.2, -0.1, -0.2, -0.2), P_7(0.1, 0.3, 0.5, -0.4, -0.5, -0.2)>} be a bipolar neutrosophic set of Z. Then from definition (2.6), we obtain

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$\underline{N}(P)$ = {<P_1(0.2, 0.5, 0.4, -0.4, -0.3, -0.1), P_4(0.2, 0.5, 0.4, -0.4, -0.3, -0.1), P_5(0.4, 0.6, 0.2, -0.1, -0.2, -0.2)}.

$\overline{N}(P)$ = {<P_1(0.3, 0.3, 0.4, -0.3, -0.3, -0.2), P_4(0.3, 0.3, 0.4, -0.3, -0.3, -0.2), P_5(0.4, 0.6, 0.2, -0.1, -0.2, -0.2), P_7(0.1, 0.3, 0.5, -0.4, -0.5, -0.2), P_8(0.1, 0.3, 0.5, -0.4, -0.5, -0.2)}.

Example 3.3

Let N(Q) = {<Q_1(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), Q_4(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), Q_5(0.4, 0.6, 0.2, -0.1, -0.2, -0.2)>} be a bipolar neutrosophic set of Z. Then we obtain lower approximation and upper approximation of N(Q) as follows:

$\underline{N}(Q)$ = {< Q_1(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), Q_4(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), Q_5(0.4, 0.6, 0.2, -0.1, -0.2, -0.2)}.

$\overline{N}(Q)$ = {< Q_1(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), Q_4(0.2, 0.3, 0.4, -0.4, -0.3, -0.1), Q_5(0.4, 0.6, 0.2, -0.1, -0.2, -0.2)}.

Obviously, $\underline{N}(Q) = \overline{N}(Q)$

Definition 3.4 Containment

Let A = $[\underline{N}(P), \overline{N}(P)]$ and B = $[\underline{N}(Q), \overline{N}(Q)]$ be two rough bipolar neutrosophic sets. Then, $A \subseteq B$ iff the following conditions hold.

$T^{+}_{\underline{N}(P)}(x) \le T^{+}_{\overline{N}(Q)}(x)$, $I^{+}_{\underline{N}(P)}(x) \le I^{+}_{\overline{N}(Q)}(x)$, $F^{+}_{\underline{N}(P)}(x) \ge F^{+}_{\overline{N}(Q)}(x)$, $T^{-}_{\underline{N}(P)}(x) \ge T^{-}_{\overline{N}(Q)}(x)$, $T^{-}_{\underline{N}(P)}(x) \ge T^{-}_{\overline{N}(Q)}(x)$ and

$T^{-}_{\underline{N}(P)}(x) \le T^{-}_{\overline{N}(Q)}(x)$, for $\forall x \in Z$.

Definition 3.5 Equality of two RBNS

Two rough bipolar neutrosophic sets $[\underline{N}(P), \overline{N}(P)]$ and $[\underline{N}(Q), \overline{N}(Q)]$ are equal if the following conditions hold.

$T^{+}_{\underline{N}(P)}(x) = T^{+}_{\overline{N}(Q)}(x)$, $I^{+}_{\underline{N}(P)}(x) = I^{+}_{\overline{N}(Q)}(x)$, $F^{+}_{\underline{N}(P)}(x) = F^{+}_{\overline{N}(Q)}(x)$, $T^{-}_{\underline{N}(P)}(x) = T^{-}_{\overline{N}(Q)}(x)$, $T^{-}_{\underline{N}(P)}(x) = T^{-}_{\overline{N}(Q)}(x)$ and

$T^{-}_{\underline{N}(P)}(x) = T^{-}_{\overline{N}(Q)}(x)$, for $\forall x \in Z$.

Definition 3.6 Union

Let A = $[\underline{N}(S), \overline{N}(S)]$ and B = $[\underline{N}(Q), \overline{N}(Q)]$ be two rough bipolar neutrosophic sets. Then, $A \cup B$ is defined as follows:

$$A \cup B(x) = \left(\begin{matrix} \left(\begin{matrix} \max(T^{+}_{\underline{N}(S)}(x), T^{+}_{\underline{N}(Q)}(x)), \dfrac{I^{+}_{\underline{N}(S)}(x) + I^{+}_{\underline{N}(Q)}(x)}{2}, \min(F^{+}_{\underline{N}(S)}(x), F^{+}_{\underline{N}(Q)}(x)), \\ \min(T^{-}_{\underline{N}(S)}(x), T^{-}_{\underline{N}(Q)}(x)), \dfrac{I^{-}_{\underline{N}(S)}(x) + I^{-}_{\underline{N}(Q)}(x)}{2}, \max(F^{-}_{\underline{N}(S)}(x), F^{-}_{\underline{N}(Q)}(x)) \end{matrix} \right), \\ \left(\begin{matrix} \max(T^{+}_{\overline{N}(S)}(x), T^{+}_{\overline{N}(Q)}(x)), \dfrac{I^{+}_{\underline{N}(S)}(x) + I^{+}_{\overline{N}(Q)}(x)}{2}, \min(F^{+}_{\overline{N}(S)}(x), F^{+}_{\overline{N}(Q)}(x)), \\ \min(T^{-}_{\overline{N}(S)}(x), T^{-}_{\overline{N}(Q)}(x)), \dfrac{I^{-}_{\overline{N}(S)}(x) + I^{-}_{\overline{N}(Q)}(x)}{2}, \max(F^{-}_{\overline{N}(S)}(x), F^{-}_{\overline{N}(Q)}(x)) \end{matrix} \right) \end{matrix} \right), \text{for } \forall x \in Z.$$

Example 3.4: Let X={x_1, x_2, x_3}. Two rough bipolar neutrosophic sets A and B on X are

$$A = \left\langle \begin{matrix} < x_1 \{(0.50, 0.30, 0.10, -0.60, -0.40, -0.08), (0.60, 0.40, 0.20, -0.50, -0.30, -0.12)\} > \\ < x_2 \{(0.30, 0.50, 0.30, -0.40, -0.40, -0.18), (0.60, 0.10, 0.20, -0.30, -0.30, -0.10)\} > \\ < x_3 \{(0.50, 0.10, 0.30, -0.25, -0.15, -0.20), (0.40, 0.40, 0.10, -0.35, -0.45, -0.20)\} > \end{matrix} \right\rangle$$

$$B = \left\langle \begin{matrix} < x_1 \{(0.60, 0.40, 0.20, -0.70, -0.50, -0.18), (0.70, 0.50, 0.30, -0.60, -0.40, -0.22)\} > \\ < x_2 \{(0.40, 0.60, 0.40, -0.50, -0.50, -0.28), (0.70, 0.20, 0.30, -0.40, -0.40, -0.20)\} > \\ < x_3 \{(0.60, 0.20, 0.40, -0.35, -0.25, -0.30), (0.50, 0.50, 0.20, -0.45, -0.55, -0.30)\} > \end{matrix} \right\rangle$$

$$\text{Now } A \cup B = \left\langle \begin{array}{l} < x_1 \{(0.60, 0.35, 0.10, -0.70, -0.45, -0.08), (0.70, 0.45, 0.20, -0.60, -0.35, -0.12)\} > \\ < x_2 \{(0.40, 0.55, 0.30, -0.50, -0.45, -0.18), (0.70, 0.15, 0.20, -0.40, -0.35, -0.10)\} > \\ < x_3 \{(0.60, 0.15, 0.30, -0.35, -0.20, -0.20), (0.50, 0.45, 0.10, -0.45, -0.50, -0.20)\} > \end{array} \right\rangle$$

Definition 3.7: Intersection:

Let A = $[\underline{N}(S), \overline{N}(S)]$ and B = $[\underline{N}(Q), \overline{N}(Q)]$ be two rough bipolar neutrosophic sets. Then, $A \cap B$ is defined as

$$A \cap B(x) = \left(\begin{array}{l} \left(\begin{array}{l} \min(T^{+}_{\underline{N}(S)}(x), T^{+}_{\underline{N}(Q)}(x)), \dfrac{I^{+}_{\underline{N}(S)}(x) + I^{+}_{\underline{N}(Q)}(x)}{2}, \max(F^{+}_{\underline{N}(S)}(x), F^{+}_{\underline{N}(Q)}(x)), \\ \max(T^{-}_{\underline{N}(S)}(x), T^{-}_{\underline{N}(Q)}(x)), \dfrac{I^{-}_{\underline{N}(S)}(x) + I^{-}_{\underline{N}(Q)}(x)}{2}, \min(F^{-}_{\underline{N}(S)}(x), F^{-}_{\underline{N}(Q)}(x)) \end{array} \right), \\ \left(\begin{array}{l} \min(T^{+}_{\overline{N}(S)}(x), T^{+}_{\overline{N}(Q)}(x)), \dfrac{I^{+}_{\underline{N}(S)}(x) + I^{+}_{\overline{N}(Q)}(x)}{2}, \max(F^{+}_{\overline{N}(S)}(x), F^{+}_{\overline{N}(Q)}(x)), \\ \max(T^{-}_{\overline{N}(S)}(x), T^{-}_{\overline{N}(Q)}(x)), \dfrac{I^{-}_{\overline{N}(S)}(x) + I^{-}_{\overline{N}(Q)}(x)}{2}, \min(F^{-}_{\overline{N}(S)}(x), F^{-}_{\overline{N}(Q)}(x)) \end{array} \right) \end{array} \right), \text{for } \forall x \in Z.$$

Example 3.5

Let X={x_1, x_2, x_3}. Two rough bipolar neutrosophic sets A and B on X are

$$A = \left\langle \begin{array}{l} < x_1 \{(0.50, 0.30, 0.10, -0.60, -0.40, -0.08), (0.60, 0.40, 0.20, -0.50, -0.30, -0.12)\} > \\ < x_2 \{(0.30, 0.50, 0.30, -0.40, -0.40, -0.18), (0.60, 0.10, 0.20, -0.30, -0.30, -0.10)\} > \\ < x_3 \{(0.50, 0.10, 0.30, -0.25, -0.15, -0.20), (0.40, 0.40, 0.10, -0.35, -0.45, -0.20)\} > \end{array} \right\rangle$$

$$B = \left\langle \begin{array}{l} < x_1 \{(0.60, 0.40, 0.20, -0.70, -0.50, -0.18), (0.70, 0.50, 0.30, -0.60, -0.40, -0.22)\} > \\ < x_2 \{(0.40, 0.60, 0.40, -0.50, -0.50, -0.28), (0.70, 0.20, 0.30, -0.40, -0.40, -0.20)\} > \\ < x_3 \{(0.60, 0.20, 0.40, -0.35, -0.25, -0.30), (0.50, 0.50, 0.20, -0.45, -0.55, -0.30)\} > \end{array} \right\rangle$$

$$\text{Now } A \cap B = \left\langle \begin{array}{l} < x_1 \{(0.50, 0.35, 0.20, -0.60, -0.45, -0.18), (0.60, 0.45, 0.30, -0.50, -0.35, -0.22)\} > \\ < x_2 \{(0.30, 0.55, 0.40, -0.50, -0.45, -0.18), (0.60, 0.15, 0.30, -0.30, -0.35, -0.20)\} > \\ < x_3 \{(0.50, 0.15, 0.40, -0.25, -0.20, -0.30), (0.40, 0.45, 0.20, -0.35, -0.50, -0.30)\} > \end{array} \right\rangle$$

Definition 3.8: Complement

If N(S) = $[\underline{N}(S), \overline{N}(S)]$ is a rough bipolar neutrosophic set in (U, R), the complement of N(S) is the rough bipolar neutrosophic set defined by

~N(S) = $[\underline{N}(S)^c, \overline{N}(S)^c]$ where, $\underline{N}(S)^c$ and $\overline{N}(S)^c$ are the complements of bipolar neutrosophic sets.

$$\underline{N}(S)^c = \left\langle < x, 1 - T^{+}_{\underline{N}(S)}(x), 1 - I^{+}_{\underline{N}(S)}(x), 1 - F^{+}_{\underline{N}(S)}(x), -1 - T^{-}_{\underline{N}(S)}(x), -1 - I^{-}_{\underline{N}(S)}(x), -1 - F^{-}_{\underline{N}(S)}(x) > / z \in [x]_R, x \in Z \right\rangle,$$

$$\overline{N}(S)^c = \left\langle < x, 1 - T^{+}_{\overline{N}(S)}(x), 1 - T^{+}_{\overline{N}(S)}(x), 1 - T^{+}_{\overline{N}(S)}(x), -1 - T^{-}_{\overline{N}(S)}(x), -1 - I^{-}_{\overline{N}(S)}(x), -1 - F^{-}_{\overline{N}(S)}(x) > / z \in [x]_R, x \in Z \right\rangle$$

Example 3.6

Let X={x_1, x_2, x_3}. A rough bipolar neutrosophic set A on X is

$$A = \left\langle \begin{array}{l} < x_1 \{(0.50, 0.30, 0.10, -0.60, -0.40, -0.08), (0.60, 0.40, 0.20, -0.50, -0.30, -0.12)\} > \\ < x_2 \{(0.30, 0.50, 0.30, -0.40, -0.40, -0.18), (0.60, 0.10, 0.20, -0.30, -0.30, -0.10)\} > \\ < x_3 \{(0.50, 0.10, 0.30, -0.25, -0.15, -0.20), (0.40, 0.40, 0.10, -0.35, -0.45, -0.20)\} > \end{array} \right\rangle$$

then the complement of A is given as follows.

$$A^c = \left\langle \begin{array}{l} < x_1 \{(0.50, 0.70, 0.90, -0.40, -0.60, -0.92), (0.40, 0.60, 0.80, -0.50, -0.70, -0.88)\} > \\ < x_2 \{(0.70, 0.50, 0.70, -0.60, -0.60, -0.82), (0.40, 0.90, 0.80, -0.70, -0.70, -0.90)\} > \\ < x_3 \{(0.50, 0.90, 0.70, -0.75, -0.85, -0.80), (0.60, 0.60, 0.90, -0.65, -0.55, -0.80)\} > \end{array} \right\rangle$$

Definition 3.9
If N(S) and N(Q) are two rough bipolar neutrosophic sets in Z ,then we define the following:
I. N(S) = N(Q) if and only if $\underline{N}(S) = \underline{N}(Q)$ and $\overline{N}(S) = \overline{N}(Q)$
II. N(S) $\subseteq$ N(Q) if and only if $\underline{N}(S) \subseteq \underline{N}(Q)$ and $\overline{N}(S) \subseteq \overline{N}(Q)$
III. N(S) $\cup$ N(Q) if and only if $< \underline{N}(S) \cup \underline{N}(Q),\ \overline{N}(S) \cup \overline{N}(Q) >$
IV. N(S) $\cap$ N(Q) if and only if $< \underline{N}(S) \cap \underline{N}(Q),\ \overline{N}(S) \cap \overline{N}(Q) >$
V. N(S) + N(Q) if and only if $< \underline{N}(S) + \underline{N}(Q),\ \overline{N}(S) + \overline{N}(Q) >$
VI. N(S).N(Q) if and only if $< \underline{N}(S) . \underline{N}(Q),\ \overline{N}(S) . \overline{N}(Q) >$

Proposition 3.1
If M, Q, S are rough bipolar neutrosophic sets in (Z, R), then the following propositions hold.
I. ~M(~M) = M
II. $\underline{N}(M) \subseteq \overline{N}(M)$
III. $<\sim (\underline{N}(M) \cup \underline{N}(Q)) = \sim (\underline{N}(M)) \cap \sim (\underline{N}(Q))$
IV. $<\sim (\underline{N}(M) \cap \underline{N}(Q)) = \sim (\underline{N}(M)) \cup \sim (\underline{N}(Q))$
V. $<\sim (\overline{N}(M) \cup \overline{N}(Q)) = \sim (\overline{N}(M)) \cap \sim (\overline{N}(Q))$
VI. $<\sim (\overline{N}(M) \cap \overline{N}(Q)) = \sim (\overline{N}(M)) \cup \sim (\overline{N}(Q))$
VII. $M \cup Q = Q \cup M$ and $M \cap Q = Q \cap M$
VIII. $M \cup (Q \cup S) = (M \cup Q) \cup S$ and $M \cap (Q \cap S) = (M \cap Q) \cap S$

Proof of I
If N(M) = $[\underline{N}(M), \overline{N}(M)]$ is a rough bipolar neutrosophic set in (U, R), the complement of N(P) is the rough bipolar neutrosophic set defined as follows.

$$\sim \underline{N}(M) = \left\langle < x,\ 1 - T^{+}_{\underline{N}(M)}(x), 1 - I^{+}_{\underline{N}(M)}(x),\ 1 - F^{+}_{\underline{N}(M)}(x), -1 - T^{-}_{\underline{N}(M)}(x), -1 - I^{-}_{\underline{N}(M)}(x),\ -1 - F^{-}_{\underline{N}(M)}(x) > / z \in [x]_R, x \in Z \right\rangle,$$

$$\sim \overline{N}(M) = \left\langle < x,\ 1 - T^{+}_{\overline{N}(M)}(x), 1 - T^{+}_{\overline{N}(M)}(x), 1 - T^{+}_{\overline{N}(M)}(x), -1 - T^{-}_{\overline{N}(M)}(x), -1 - I^{-}_{\overline{N}(M)}(x), -1 - F^{-}_{\overline{N}(M)}(x) > / z \in [x]_R, x \in Z \right\rangle$$

From this definition it is obvious that, ~M(~M) = M.

Proof of II
The lower and the upper approximations of M in the approximation (Z, R) denoted by $\underline{N}(M)$ and $\overline{N}(M)$ are respectively defined as follows:

$$\underline{N}(M) = \left\langle < x,\ T^{+}_{\underline{N}(M)}(x), I^{+}_{\underline{N}(M)}(x),\ F^{+}_{\underline{N}(M)}(x), T^{-}_{\underline{N}(M)}(x), I^{-}_{\underline{N}(M)}(x),\ F^{-}_{\underline{N}(M)}(x) > / z \in [x]_R, x \in Z \right\rangle,$$

$$\overline{N}(M) = \left\langle < x,\ T^{+}_{\overline{N}(M)}(x), T^{+}_{\overline{N}(M)}(x), T^{+}_{\overline{N}(M)}(x), T^{-}_{\overline{N}(M)}(x), T^{-}_{\overline{N}(M)}(x), T^{-}_{\overline{N}(M)}(x) > / z \in [x]_R, x \in Z \right\rangle$$

Where, $T^{+}_{\underline{N}(M)}(x) = \wedge_z \in [x]_R\ T^{+}(z),\ I^{+}_{\underline{N}(M)}(x) = \wedge_z \in [x]_R\ I^{+}(z),\ F^{+}_{\underline{N}(M)}(x) = \wedge_z \in [x]_R\ F^{+}(z),$

$T^{-}_{\underline{N}(M)}(x) = \wedge_z \in [x]_R\ T^{-}(z),\ I^{-}_{\underline{N}(M)}(x) = \wedge_z \in [x]_R\ I^{-}(z),\ F^{-}_{\underline{N}(M)}(x) = \wedge_z \in [x]_R\ F^{-}(z);$

$T^{+}_{\overline{N}(M)}(x) = \vee_z \in [x]_R\ T^{+}(z)\ , I^{+}_{\overline{N}(M)}(x) = \vee_z \in [x]_R\ I^{+}(z),\ F^{+}_{\overline{N}(M)}(x) = \vee_z \in [x]_R\ F^{+}(z),$

$T^{-}_{\overline{N}(M)}(x) = \vee_z \in [x]_R\ T^{-}(z)\ , I^{-}_{\overline{N}(M)}(x) = \vee_z \in [x]_R\ I^{-}(z)\quad F^{-}_{\overline{N}(M)}(x) = \vee_z \in [x]_R\ F^{-}(z).$

So, $\underline{N}(M) \subseteq \overline{N}(M)$

Proof of III

Assume that $x \in \sim\left(\underline{N}(M) \cup \underline{N}(Q)\right)$

$\Rightarrow x \in \sim \underline{N}(M)$ and $x \in \sim \underline{N}(Q)$

$\Rightarrow x \in \sim\left(\underline{N}(M)\right) \cap \sim\left(\underline{N}(Q)\right)$

$\Rightarrow x \in \sim\left(\underline{N}(M)\right) \cap \sim\left(\underline{N}(Q)\right)$

$\Rightarrow \sim\left(\underline{N}(M) \cup \underline{N}(Q)\right) \subseteq \sim\left(\left(\underline{N}(M)\right) \cap \sim\left(\underline{N}(Q)\right)\right)$

Similarly, $\sim\left(\underline{N}(M) \cup \underline{N}(Q)\right) \supseteq \sim\left(\left(\underline{N}(M)\right) \cap \sim\left(\underline{N}(Q)\right)\right)$

Hence, $\sim\left(\underline{N}(M) \cup \underline{N}(Q)\right) = \sim\left(\left(\underline{N}(M)\right) \cap \sim\left(\underline{N}(Q)\right)\right)$

Proof of IV

Assume that $x \in \sim\left(\underline{N}(M) \cap \underline{N}(Q)\right)$

$\Rightarrow x \in \sim \underline{N}(M)$ or $x \in \sim \underline{N}(Q)$

$\Rightarrow x \in \sim\left(\underline{N}(M)\right) \cup \sim\left(\underline{N}(Q)\right)$

$\Rightarrow x \in \sim\left(\underline{N}(M)\right) \cup \sim\left(\underline{N}(Q)\right)$

$\Rightarrow \sim\left(\underline{N}(M) \cap \underline{N}(Q)\right) \subseteq \sim\left(\left(\underline{N}(M)\right) \cup \sim\left(\underline{N}(Q)\right)\right)$

Similarly, $\sim\left(\underline{N}(M) \cap \underline{N}(Q)\right) \supseteq \sim\left(\left(\underline{N}(M)\right) \cup \sim\left(\underline{N}(Q)\right)\right)$

Hence, $\sim\left(\underline{N}(M) \cap \underline{N}(Q)\right) = \sim\left(\left(\underline{N}(M)\right) \cup \sim\left(\underline{N}(Q)\right)\right)$

Proof of V

Assume that $x \in \sim\left(\overline{N}(M) \cup \overline{N}(Q)\right)$

$\Rightarrow x \in \sim \overline{N}(M)$ and $x \in \sim \overline{N}(Q)$

$\Rightarrow x \in \sim\left(\overline{N}(M)\right) \cap \sim\left(\overline{N}(Q)\right)$

$\Rightarrow x \in \sim\left(\overline{N}(M)\right) \cap \sim\left(\overline{N}(Q)\right)$

$\Rightarrow \sim\left(\overline{N}(M) \cup \overline{N}(Q)\right) \subseteq \sim\left(\left(\overline{N}(M)\right) \cap \sim\left(\overline{N}(Q)\right)\right)$

Similarly, $\sim\left(\overline{N}(M) \cup \overline{N}(Q)\right) \supseteq \sim\left(\left(\overline{N}(M)\right) \cap \sim\left(\overline{N}(Q)\right)\right)$

Hence, $\sim\left(\overline{N}(M) \cup \overline{N}(Q)\right) = \sim\left(\left(\overline{N}(M)\right) \cap \sim\left(\overline{N}(Q)\right)\right)$

Proof of VI

Assume that $x \in \sim\left(\overline{N}(M) \cap \overline{N}(Q)\right)$

$\Rightarrow x \in \sim \overline{N}(M)$ or $x \in \sim \overline{N}(Q)$

$\Rightarrow x \in \sim\left(\overline{N}(M)\right) \cup \sim\left(\overline{N}(Q)\right)$

$\Rightarrow x \in \sim\left(\overline{N}(M)\right) \cup \sim\left(\overline{N}(Q)\right)$

$\Rightarrow \sim\left(\overline{N}(M) \cap \overline{N}(Q)\right) \subseteq \sim\left(\left(\overline{N}(M)\right) \cup \sim\left(\overline{N}(Q)\right)\right)$

Similarly, $\sim\left(\overline{N}(M) \cap \overline{N}(Q)\right) \supseteq \sim\left(\left(\overline{N}(M)\right) \cup \sim\left(\overline{N}(Q)\right)\right)$

Hence, $\sim\left(\overline{N}(M) \cap \overline{N}(Q)\right) = \sim\left(\left(\overline{N}(M)\right) \cup \sim\left(\overline{N}(Q)\right)\right)$

Proofs of VII and VIII are obvious.

Proof of VII.

1st part:

Assume that $x \in M \cup Q$ then, $x \in Q \cup M$

$\Rightarrow M \cup Q \subseteq Q \cup M$

Again, consider $y \in Q \cup M$ then, $y \in M \cup Q$

$\Rightarrow Q \cup M \subseteq M \cup Q$

$\Rightarrow M \cup Q \supseteq Q \cup M$

Hence, $M \cup Q = Q \cup M$

2nd part:

Assume that $x \in M \cap Q$ then, $x \in Q \cap M$

$\Rightarrow M \cap Q \subseteq Q \cap M$

Again, consider $y \in Q \cap M$ then, $y \in M \cap Q$

$\Rightarrow Q \cap M \subseteq M \cap Q$

$\Rightarrow M \cap Q \supseteq Q \cap M$

Hence, $M \cap Q = Q \cap M$

Proof of VIII

1st part:

Assume that $x \in M \cup (Q \cup S)$ then, $x \in (M \cup Q) \cup S$

$\Rightarrow M \cup (Q \cup S) \subseteq (M \cup Q) \cup S$

Again, Assume that $y \in (M \cup Q) \cup S$ then, $y \in M \cup (Q \cup S)$

$\Rightarrow M \cup (Q \cup S) \supseteq (M \cup Q) \cup S$

Hence, $M \cup (Q \cup S) = (M \cup Q) \cup S$

2nd part:

Assume that $x \in M \cap (Q \cap S)$ then, $x \in (M \cap Q) \cap S$

$\Rightarrow M \cap (Q \cap S) \subseteq (M \cap Q) \cap S$

Again, Assume that $y \in (M \cap Q) \cap S$ then, $y \in M \cap (Q \cap S)$

$\Rightarrow M \cap (Q \cap S) \supseteq (M \cap Q) \cap S$

Hence, $M \cap (Q \cap S) = (M \cap Q) \cap S$

Proposition 3.2

I. $\sim [N(P) \cup N(Q)] = (\sim N(P)) \cap (\sim N(Q))$

II. $\sim [N(P) \cap N(Q)] = (\sim N(P)) \cup (\sim N(Q))$

Proof of I

$\sim [N(P) \cup N(Q)]$

$= \sim < \underline{N}(P) \cup \underline{N}(Q), \overline{N}(P) \cup \overline{N}(Q) >$

$= <\sim \left(\underline{N}(P) \cap \underline{N}(Q)\right), \sim \left(\overline{N}(P) \cap \overline{N}(Q)\right)>$

$= (\sim N(P)) \cap (\sim N(Q))$

Proof of II

$\sim [N(P) \cap N(Q)]$

$= \sim < \underline{N}(P) \cap \underline{N}(Q), \overline{N}(P) \cap \overline{N}(Q) >$

$= <\sim \left(\underline{N}(P) \cup \underline{N}(Q)\right), \sim \left(\overline{N}(P) \cup \overline{N}(Q)\right)>$

$= (\sim N(P)) \cup (\sim N(Q))$

Proposition 3.3

If S and Q are two rough bipolar neutrosophic sets such that $S \subseteq Q$ implies $N(S) \subseteq N(Q)$ then,

I. $N(S \cup Q) \supseteq N(S) \cup N(Q)$

II. $N(S \cap Q) \supseteq N(S) \cap N(Q)$

Proof of I

$$T^{+}_{\underline{N}(S \cup Q)}(x) = \inf \left[T^{+}_{(S \cup Q)}(x) \right] : \forall x \in X$$

$= \inf\left[\max T_S^+(x), T_Q^+(x)\right] : \forall x \in X$

$\geq \max\left[\inf T_S^+(x), \inf T_Q^+(x)\right] : \forall x \in X$

$= \max\left[T^+_{\underline{N}(S)}(x), T^+_{\underline{N}(Q)}(x)\right] : \forall x \in X$

$= T^+_{\underline{N}(S)}(x) \cup T^+_{\underline{N}(Q)}(x) : \forall x \in X$

Similarly,

$I^+_{\underline{N}(S\cup Q)}(x) \leq I^+_{\underline{N}(S)}(x) \cup I^+_{\underline{N}(Q)}(x) : \forall x \in X$

$F^+_{\underline{N}(S\cup Q)}(x) \leq F^+_{\underline{N}(S)}(x) \cup F^+_{\underline{N}(Q)}(x) : \forall x \in X$

$T^-_{\underline{N}(S\cup Q)}(x) \leq T^-_{\underline{N}(S)}(x) \cup T^-_{\underline{N}(Q)}(x) : \forall x \in X$

$I^-_{\underline{N}(S\cup Q)}(x) \geq I^-_{\underline{N}(S)}(x) \cup I^-_{\underline{N}(Q)}(x) : \forall x \in X$

$F^-_{\underline{N}(S\cup Q)}(x) \geq F^-_{\underline{N}(S)}(x) \cup F^-_{\underline{N}(Q)}(x) : \forall x \in X$

Thus, $\underline{N}(S \cup Q) \supseteq \underline{N}(S) \cup \underline{N}(Q))$

We can also see that, $\overline{N}(S \cup Q) \supseteq \overline{N}(S) \cup \overline{N}(Q)$

Hence, $N(S \cup Q) \supseteq N(S) \cup N(Q)$

Proof of II

$T^+_{\underline{N}(S\cap Q)}(x) = \sup\left[T^+_{(S\cap Q)}(x)\right] : \forall x \in U$

$= \sup\left[\min T_S^+(x), T_Q^+(x)\right] : \forall x \in U$

$\geq \min\left[\sup T_S^+(x), \sup T_Q^+(x)\right] : \forall x \in U$

$= \min\left[T^+_{\underline{N}(S)}(x), T^+_{\underline{N}(Q)}(x)\right] : \forall x \in U$

$= T^+_{\underline{N}(S)}(x) \cap T^+_{\underline{N}(Q)}(x) : \forall x \in U$

Similarly,

$I^+_{\underline{N}(S\cap Q)}(x) \leq I^+_{\underline{N}(S)}(x) \cap I^+_{\underline{N}(Q)}(x) : \forall x \in U,$

$F^+_{\underline{N}(S\cap Q)}(x) \leq F^+_{\underline{N}(S)}(x) \cap F^+_{\underline{N}(Q)}(x) : \forall x \in U,$

$T^-_{\underline{N}(S\cap Q)}(x) \leq T^-_{\underline{N}(S)}(x) \cap T^-_{\underline{N}(Q)}(x) : \forall x \in U,$

$I^-_{\underline{N}(S\cap Q)}(x) \geq I^-_{\underline{N}(S)}(x) \cap I^-_{\underline{N}(Q)}(x) : \forall x \in U,$

$F^-_{\underline{N}(S\cap Q)}(x) \geq F^-_{\underline{N}(S)}(x) \cap F^-_{\underline{N}(Q)}(x) : \forall x \in U,$

Thus, $\underline{N}(S \cap Q) \supseteq \underline{N}(S) \cap \underline{N}(Q))$

We can also see that $\overline{N}(S \cap Q) \supseteq \overline{N}(S) \cap \overline{N}(Q)$

Hence, $N(S \cup Q) \supseteq N(S) \cup N(Q)$.

CONCLUSIONS

In this paper we introduce the concept of rough bipolar neutrosophic set. We also study some properties on them and prove some propositions. The concept combines two different concepts namely, neutrosophic rough set and bipolar neutrosophic set. While bipolar neutrosophic set theory is mainly concerned with, indeterminate and inconsistent information, rough set theory is with incompleteness; but both the theories deal with imprecision. It is clear that rough bipolar neutrosophic sets can be utilized for dealing with both of indeterminacy and incompleteness.The proposed concept can be used in practical decision making roblem.

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